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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

98. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A poor man borrowed \$20 which he repaid in eleven monthly installments of \$2 each; what was the annual rate of interest (reckoned as simple interest)?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Natural Science, Chester High School, Chester, Pa.

Here we use the same formula we have used often before in the MONTHLY:

$$p=\frac{Pr(1+r)^n}{(1+r)^n-1}.$$

Where $p=2$, $P=20$, $n=11$.

$\therefore 2(1+r)^{11}-2=20r(1+r)^{11}$. $\therefore 2(1+r)^{11}(1-10r)=2$.

$\therefore r=.016$ nearly. $12r=.192=19\frac{1}{5}\%$ nearly.

$\therefore 19\frac{1}{5}\%$=rate of interest.

Also solved by *ELMER SCHUYLER*.

99. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

If 300 cats kill 300 rats in 300 minutes, how many cats will kill 100 rats in 100 minutes?

I. Solution by the PROPOSER.

1. If 300 cats kill 300 rats in 300 minutes,
2. 1 cat will kill 1 rat in 300 minutes,
3. 1 cat will kill 100 rats in 3000 minutes, and
4. 300 cats will kill 100 rats in 100 minutes.

II. Solution by CHARLES C. CROSS, Libertytown, Md.

If 300 cats catch 300 rats in 300 minutes, then 1 cat will catch 300 rats in 9000 minutes, or 1 cat will catch 100 rats in 3000 minutes.

Hence 300 cats will catch 100 rats in 100 minutes.

Also solved by *G. B. M. ZERR*, *FREMONT CRANE*, and *ALOIS F. KAVORIK*.

ALGEBRA.

86. Proposed by J. MARCUS BOORMAN, Consultative Mechanician, and Counsellor at Law, Woodmere, Long Island, N. Y.

Solve $x^2+yz=16 \ldots\ldots(A)$; $y^2+xz=17 \ldots\ldots(B)$; $z^2+xy=22 \ldots\ldots(C)$, for all the roots.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science in Chester High School, Chester, Pa., and Prof. J. SCHEFFER, A. M., Hagerstown, Md.

Let $y=vx$, $z=wx$.

$\therefore x^2+vwx^2=16$, $v^2x^2+wx^2=17$, $w^2x^2+vx^2=22 \ldots\ldots(1, 2, 3)$.

(1)÷(2), $(1+vw)\div(v^2+w)=\frac{16}{17}$......(4).

(1)÷(3), $(1+vw)\div(w^2+v)=\frac{8}{11}$......(5).

Eliminating v between (4) and (5), we get

$59w^4-378w^3+704w^2-315w-106=0$......(6).

$\therefore\ w=2$, or $-.21774$, or $2.81226\pm.17307\sqrt{(-1)}$.

$v=1.5$, or 1.02072, or $2.29118\pm.14936\sqrt{(-1)}$.

$x=2$, or 4.53566, or

$y=3$, or 4.62964,

$z=4$, or $-.98759$.

v is found from (5). $\therefore\ v=(8w^2-11)\div(11w-8)$.

The imaginary values of x, y, and z can be found from $x=4\div\sqrt{(1+vw)}$, $y=vx$, $z=wx$.

[The only rational value of (6) is $w=2$. $\therefore\ v=\frac{3}{2}$, and substituting $z=2x$, $y=3x\div2$ in the first of the given equations we get $4x^2=16$, whence $x=\pm2$, $y=\pm3$, $z=\pm4$. Dividing the biquadratic by $w-2$, we get the cubic equation $59w^3-260w^2+184w+73=0$, which furnishes the other roots of w, these being, however, irrational. SCHEFFER.]

II. Solution by the PROPOSER.

By r (ratio) $y=rx$: ..... $z^2+rx^2=22$......(C).

$x^2+rxz=16$......(A); deduct $r(B)-(A)$.

$\therefore\ r^2x^2+xz=17$......(B); $(r^3-1)x^2=17r-15$......(D).

Deduct $r^2(A)-(B)$ ....$(r^3-1)xz=16r^2-17$......(E).

$(r^3-1)(C)$ is......$(r^4-r)x^2+(r^3-1)z^2=22r^3-22$.

$r'(D)$ is..........$(r^4-r)x^2$..........$=17r^2-16r$. Subtracting,

$$(r^3-1)z^2=22r^3-17r^2+16r-22.$$

Multiplying this equation by (D),

$$(r^3-1)^2x^2z^2=374r^3-641r^3+544r^2-630r+352.$$

$$(r^3-1)^2x^2z^2=256r^4-544r^2+289.\quad\text{Subtracting }(E)^2,$$

$$0=118r^4-641r^3+1088r^2-630r+63\ \ldots\ldots\ldots(E)^a$$

Obviously $r=1.5$. $\therefore$ 177 − 696 588 − 63

$$\therefore\ 118r^3-464r^2+392r-42=0\ldots\ldots(E_1).$$

$\frac{1}{2}(E_1)$ is..........$59r^3-232r^2+196r-21=0$......(F) $\therefore\ r=1.5$.

$r_1=0.125069\ 382246\ 530323\ 732\pm$

$r_2=1.021700\ 042125\ 541767\ 819\pm$

$r_a=2.785433\ 965458\ 436383\ 025$. *Same way*, $z=tx$.

$\therefore\ 59t^3-260t^2+184t+53=0$......(G). *Obviously*, $t=2$.

$t_1=\ 1.207289\ 829921\ 583026\ 2730\pm$

$t_2=-0.217739\ 956753\ 32493$ .....

$t_a=\ 3.417229\ 787848\ 69105$...... *Also got by*

$(E)\div(D)=t=(16r^2-17)\div(17r-16)$...... (H) ......is $[16(1.5)^2-17]\div[17(1.5)-16]=2=t$, etc.

Get x by (A)......$x^2(1+rt)=16$; *obviously* $rt=\frac{3}{2}(2)=3$.

$\therefore x^2(1+3)=16$; ......$x=\pm_1 2$; $y=\pm_1 3$; $z=\pm_1 4$.

Test r_2......t_1, etc., by $(B)(C)$ gives $17(t^2+r)=22(r^2+t)$......(J).

Thus $r_1=0.125$......in (J) *fixes* $t_1=1.2073$ (4 true decimals).

$r_2=1.0217$ in (J) *fixes* $t_2=0.217739$ (6 true decimals).

$r_a=2.785434$ in (J) *fixes* $t_a=3.417229$ (6 true decimals).

$\therefore x_1=\pm_1$......to	$y_1=\pm_1$......to	$z_1=\pm_1$ .....
3.728406 657543 8	0.466309 516672 3	4.501267 432220 0
$x_2=\pm_1$......to	$y_2=\pm_1$......to	$z_2=\pm_1$......
4.536281 490681 969	4.634718 990123 0830	0.987729 704967 0174
$x_a=\pm_1$......to	$y_a=\pm_1$......to	$z_a=\pm_1$......
1.233342 644210 0772	3.435894 472231.4779	4.214615 222418.7456

Read $\pm_1$......$\pm_1$......$\pm_1$ "change in unison," not $\pm$ *prime*, $\mp$ *prime*. One page finds *all* 24 roots of x, y, z. Q. V. D.

[*Prof. Charles C. Cross* remarks that two solutions are given in the *Mathematical Diary* for 1831, pages 150-151, and that there is quite an interesting history of this "curious and important question" given by "Unicorn," of North Carolina.

87. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

A starts to travel around a circular island at a given point and travels at the rate of 5 miles in 4 hours. One half hour after A, B starts from a point directly opposite from A and travels in an opposite direction at the rate of 4 miles in 3 hours. One hour afterwards C starts from the same point as A and travels in an opposite direction to A at the rate of 3 miles in 2 hours. One half hour afterwards D starts from the same point as B and travels in an opposite direction to B at the rate of 2 miles in 1 hour. Required the size of the island, and when they will all be together, and how far each will have traveled at the accomplishment of this event.

Solution by the PROPOSER.

This problem furnishes a good example in indeterminate analysis. Of course an unlimited number of answers may be obtained each of which will satisfy the conditions of the problem ; but we shall be content to show in what manner one set may be obtained.

A's rate of travel equals $1\frac{1}{4}$ miles per hour, B's rate equals $1\frac{1}{3}$ miles per hour, C's rate equals $1\frac{1}{2}$ miles per hour, and D's rate equals 2 miles per hour. B starts $\frac{1}{2}$ hour after A starts, C starts $1\frac{1}{2}$ hours after A, and D starts 2 hours after A.

Let c=the circumference of the island; then A and B will be together for first time after A starts in $[(\frac{1}{2}c-1\frac{1}{4}\times\frac{1}{2})/(1\frac{1}{4}+1\frac{1}{3})]+\frac{1}{2}=(6c+8)/31$ hours.

For the second time in $[(1\frac{1}{2}c-1\frac{1}{4}\times\frac{1}{2})/(1\frac{1}{4}+1\frac{1}{3})]+\frac{1}{2}=(18c+8)/31$ hours ; and for the $(m+1)$th time in $[(6+12m)c+8]/31$ hours......(A).

A and C will be together for the first time after A starts in $[(c-1\frac{1}{2}\times1\frac{1}{4})/(1\frac{1}{2}+1\frac{1}{4})]+1\frac{1}{2}=(4c+9)/11$ hours.

For the second time in $[(2c-1\frac{1}{2}\times1\frac{1}{4})/(1\frac{1}{2}+1\frac{1}{4})]+1\frac{1}{2}=(8c+9)/11$ hours ; and for the $(n+1)$th time in $[(4+4n)c+9]/11$ hours......(B).